**Finalizing Scalaron Entropy and Twistor Formalism (RFT 9.7)**

**Scalaron Entropy Functional**

We define the scalaron’s **entropy functional** $S(t)$ by:

S(t)  =  −∫d3x  ρ(x,t) ln⁡Fc(x,t) .S(t) \;=\; -\int d^3x \; \rho(x,t)\, \ln F\_c(x,t) \,. S(t)=−∫d3xρ(x,t)lnFc​(x,t).

This formula measures how much phase *disorder* (decoherence) the scalar field has accumulated at time $t$. By construction, if the field is fully coherent so that $F\_c(x,t)=1$ everywhere, then $\ln F\_c = 0$ and **$S(t)=0$**, the minimum entropy (a pure-state configuration). As decoherence grows and $F\_c(x,t)$ falls below 1, the integrand $-\rho \ln F\_c$ becomes positive, making $S(t)$ increase. In plain terms, $S(t)$ quantifies the information lost from a perfectly ordered phase configuration – higher $S$ means the scalaron’s phase is more scrambled.

**Defining $F\_c(x,t)$:** The factor $F\_c(x,t)$ represents the **coherence fraction** at position $x$, i.e. how much of the local scalaron density still “remembers” a global phase reference. We can define $F\_c(x,t)$ in two equivalent ways useful for simulations:

* *Phase-correlation definition:* One approach is to use spatial phase correlations. For example, define a normalized complex correlation

C(x,t)  =  1M ∫d3x′  ⟨Ψ∗(x,t) Ψ(x′,t)⟩ ,C(x,t) \;=\; \frac{1}{M}\,\int d^3x'\;\langle \Psi^\*(x,t)\,\Psi(x',t)\rangle\,,C(x,t)=M1​∫d3x′⟨Ψ∗(x,t)Ψ(x′,t)⟩,

where $M=\int \rho(x')d^3x'$ is the total scalaron “mass” (or particle number) and $\Psi(x,t)$ is the scalar field (wavefunction). $C(x,t)$ effectively measures the **coherent amplitude** at $x$ by summing interference from the field at all other points. We then set

Fc(x,t)=∣C(x,t)∣ρ(x,t)/M ,F\_c(x,t) = \frac{\big|C(x,t)\big|}{\rho(x,t)/M} \,,Fc​(x,t)=ρ(x,t)/M​C(x,t)​​,

which is the magnitude of the phase-aligned part of the field at $x$, normalized by the local density. If the phases at $x$ and elsewhere are all aligned, $C(x)$ will equal $\rho(x)/M$ (since $\Psi$ can be factored out) giving $F\_c=1$. If phases are uncorrelated, the contributions cancel and $|C(x)|$ is small, yielding $F\_c \ll 1$. In practice, one can restrict the integral to a correlation length scale (phases far beyond the coherence length contribute only noise). This definition uses **phase correlation integrals** to quantify local coherence.

* *Eigenmode overlap definition:* An equivalent, simulation-friendly definition uses the **one-particle density matrix** $\hat{\rho}\_1(x,x';t) = \langle \Psi^\*(x,t)\Psi(x',t)\rangle$. Diagonalizing $\hat{\rho}\_1$ (Penrose–Onsager criterion​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx), we obtain orthonormal eigenmodes $\phi\_i(x,t)$ with eigenvalues $n\_i(t)$ (interpretable as occupation numbers or mass in mode $i$​file-3zh15rq3mb1bnnjszwe2yx). The **dominant eigenmode** $\phi\_0(x,t)$ (with eigenvalue $n\_0$) represents the largest coherent component (the condensate mode if one exists). We can then define $F\_c(x,t)$ as the fractional contribution of this dominant mode to the density at $x$:

Fc(x,t)  =  n0(t) ∣ϕ0(x,t)∣2ρ(x,t) ,F\_c(x,t) \;=\; \frac{n\_0(t)\,|\phi\_0(x,t)|^2}{\rho(x,t)}\,,Fc​(x,t)=ρ(x,t)n0​(t)∣ϕ0​(x,t)∣2​,

where $\rho(x,t)=\hat{\rho}\_1(x,x;t)=\sum\_i n\_i |\phi\_i(x)|^2$ is the total density. By construction, $0 \le F\_c(x)\le 1$. If the scalaron is mostly in a single coherent state, $n\_0$ will be close to the total mass $M$ and $\phi\_0$ will match the density profile, so $F\_c(x)\approx 1$ everywhere (near-pure state). If the field has fragmented into many incoherent modes, $n\_0 \ll M$ and $F\_c(x)$ drops. This provides a **simulation-usable measure**: one can compute $\phi\_0$ and $n\_0$ from the wavefunction or density matrix at each timestep and then obtain $S(t)$ via the above integral. In summary, $S(t)$ captures how far the scalaron has progressed from a single-wave (zero entropy) state toward a mixed-state (higher entropy) regime.

*Interpretation:* $S(t)$ increases as the scalaron’s phase coherence is lost. Early on, in a halo core or cosmic void, $\phi\_0$ contains almost all the mass (coherence fraction near 100%), so $S\approx0$. In a violently disturbed region (e.g. after mergers or in a dense cluster halo), many modes share the mass and phases are randomized, pushing $S(t)$ upward. For example, simulations show small halos can retain ~50% of mass in a coherent ground-state core, whereas large cluster halos fall below 0.1%​file-3zh15rq3mb1bnnjszwe2yx – correspondingly, $S(t)$ is near zero in the former and very large in the latter, reflecting the growth of disorder in the field.

**von Neumann Entropy Comparison**

In quantum terms, the scalaron’s coherence can also be quantified by the **von Neumann entropy** of the one-particle density matrix. Let $\hat{\rho}\_1(t)$ have eigenvalues $p\_i(t)=n\_i/M$ normalized as probabilities ($\sum\_i p\_i=1$). The von Neumann entropy is

SvN(t)  =  − Tr ⁣[ρ^1ln⁡ρ^1]  =  −∑ipi(t) ln⁡pi(t) ,S\_{\mathrm{vN}}(t) \;=\; -\,\mathrm{Tr}\!\big[\hat{\rho}\_1 \ln \hat{\rho}\_1\big] \;=\; -\sum\_i p\_i(t)\,\ln p\_i(t)\,,SvN​(t)=−Tr[ρ^​1​lnρ^​1​]=−i∑​pi​(t)lnpi​(t),

which is simply the Shannon entropy of the mode occupation probabilities​[en.wikipedia.org](https://en.wikipedia.org/wiki/Von_Neumann_entropy#:~:text=the%20von%20Neumann%20entropy%20is,j%7D%5Cleft%7Cj%5Cright%5Crangle%20%5Cleft%5Clangle%20j%5Cright%7C%2C%7D%20then)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Von_Neumann_entropy#:~:text=basis%20of%20its%20eigenvectors%20Image%3A,2). $S\_{\mathrm{vN}}=0$ indicates a pure state (only one $p\_i=1$), while larger $S\_{\mathrm{vN}}$ indicates a more mixed state. The **scalaron entropy** $S(t)$ defined above is closely related. In fact, if $F\_c(x,t)$ is defined via the dominant eigenmode fraction, one can show:

* In the limit of **gradual decoherence** (one mode dominant with small admixtures), $S(t)$ is approximately proportional to $S\_{\mathrm{vN}}(t)$. When $p\_0 \approx 1$ and $p\_{1,2,\dots}\ll 1$, we can expand $S\_{\mathrm{vN}} \approx -,(1-\epsilon)\ln(1-\epsilon) - \epsilon \ln(\epsilon)$ for $\epsilon = 1-p\_0 \ll 1$. To leading order $S\_{\mathrm{vN}}\sim \epsilon$, and meanwhile $F\_c(x)\approx p\_0$ nearly uniform in space, giving $S(t) \approx -\ln p\_0 \sim \epsilon$ as well. Thus both entropies rise in step from 0.
* More generally, $S(t)$ and $S\_{\mathrm{vN}}(t)$ track each other whenever coherence is lost *uniformly*. If the subdominant modes have a similar spatial profile as the dominant mode (so that $F\_c(x)$ is roughly constant across $x$), then $S(t) = -\ln p\_0$ (since $\rho(x)$ and $|\phi\_0(x)|^2$ cancel out in the integral) and $S\_{\mathrm{vN}} = -[p\_0 \ln p\_0 + (1-p\_0)\ln(1-p\_0)]$. In the regime $p\_0 > 0.5$, these two quantities are monotonically related (both increase as $p\_0$ decreases). Thus, **in slowly decohering, near-equilibrium scenarios, $S(t)$ is a reliable proxy for $S\_{\mathrm{vN}}$** (differing only by a factor of order unity). Both entropies capture the same trend: as the scalaron field fragments into multiple modes, the entropy increases. Only in highly heterogeneous decoherence (where $F\_c(x)$ varies wildly across space) might $S(t)$ deviate, since it weights regions by density. In practice, we expect $S(t)$ to qualitatively mirror the von Neumann entropy of the field’s one-body state during most of the evolution.

*(In summary, $S\_{\mathrm{vN}}(t)$ provides a basis-independent, global entropy measure, while $S(t)$ is a spatially-resolved entropy that gives more weight to high-density regions losing coherence. In the simulations’ gradual decoherence regimes, these two measures are proportional, reflecting the same underlying increase in disorder.)*

**Twistor Evolution Operator $\mathcal{F}$**

To incorporate the **twistor formalism**, we seek an evolution equation for the scalaron’s twistor-space representation. Let $Z$ denote coordinates in twistor space (e.g. homogeneous coordinates for a projective twistor, or other suitable labels), and $f(Z,t)$ represent the scalaron’s state in twistor space (for example, a holomorphic function or distribution whose poles/zeros encode the field configuration). We propose a general form:

∂t[f(Z,t)]  =  F([f(Z,t)],  Tμν(x,t),  ∇Zf) ,\partial\_t [f(Z,t)] \;=\; \mathcal{F}\Big( [f(Z,t)],\; T\_{\mu\nu}(x,t),\; \nabla\_Z f \Big)\,,∂t​[f(Z,t)]=F([f(Z,t)],Tμν​(x,t),∇Z​f),

where $T\_{\mu\nu}(x,t)$ is the stress-energy tensor in spacetime (which influences the twistor dynamics), and $\nabla\_Z$ denotes differentiation with respect to twistor coordinates. This **twistor evolution operator** $\mathcal{F}$ encapsulates how gravitational interactions ($T\_{\mu\nu}$) and field self-interactions drive changes in the twistor function $f(Z)$ over time. We do not require the reader to have deep familiarity with twistor theory here; essentially $f(Z)$ is an alternate way to describe the scalaron field, and $\mathcal{F}$ describes its time evolution in that representation.

For clarity, we present two **toy-model** forms of $\mathcal{F}$, illustrating different physical effects:

* **Linear gradient-driven decoherence:** In this simple model, decoherence is modeled as a diffusion or damping of the twistor function, smoothing out fine-scale phase structure. For example, one may choose

∂tf(Z)  =  D ∇Z2f(Z)  −  Γ(Z) f(Z) ,\partial\_t f(Z) \;=\; D\,\nabla^2\_Z f(Z) \;-\; \Gamma(Z)\, f(Z)\,,∂t​f(Z)=D∇Z2​f(Z)−Γ(Z)f(Z),

where $D$ is a small diffusion constant in twistor space and $\Gamma(Z)$ a damping rate that could be made proportional to local stress $T\_{\mu\nu}$ (so that regions of twistor space corresponding to high-density, rapidly evolving regions in spacetime decohere faster). This linear operator $\mathcal{F}\_\text{lin} = D \nabla^2\_Z - \Gamma$ causes any sharp features in $f(Z)$ (which correspond to well-defined phase relationships) to spread and decay over time. Physically, it represents **phase mixing**: twistor space points diffuse, eroding coherent structures. For instance, if $f(Z)$ initially has a sharp peak (a well-localized state in twistor space), diffusion will broaden that peak, representing the loss of precise phase alignment (decoherence). Such a linear model ensures mathematical simplicity and can be used to simulate gradual decoherence as a smooth process.

* **Nonlinear pole-splitting & self-interaction (with collapse):** To capture the nonlinear and potentially abrupt aspects of quantum collapse, we introduce a model where $\mathcal{F}$ contains **nonlinear terms** that can spawn additional structure in $f(Z)$ and drive rapid changes. A generic example is:

∂tf(Z)  =  v (Z−Z0) ∂Zf(Z)  −  μ ∣f(Z)∣2 f(Z) .\partial\_t f(Z) \;=\; v\, (Z-Z\_0)\,\partial\_Z f(Z)\;-\; \mu\, |f(Z)|^2\,f(Z)\,. ∂t​f(Z)=v(Z−Z0​)∂Z​f(Z)−μ∣f(Z)∣2f(Z).

Here the first term (with coefficient $v$) is a **shear in twistor space** that depends on $(Z-Z\_0)$, the offset from some reference twistor $Z\_0$. This term can stretch and split features: intuitively, if $f(Z)$ initially has a single pole at $Z\_0$, the shear term will move portions of $f$ at $Z$ slightly above and below $Z\_0$ in opposite directions, effectively **splitting one pole into two** moving poles (a simple way to model branch splitting). The second term $-\mu |f|^2 f$ is a **nonlinear damping** that kicks in strongly when $|f|^2$ (related to the density of the field in phase space) grows large – it represents a self-interaction or collapse tendency. For example, if $f(Z)$ starts to concentrate (simulating a collapsing wavefunction or forming a singularity), the $|f|^2 f$ term rapidly suppresses it (since it is cubic in amplitude, large $f$ triggers a large negative feedback), mimicking wavefunction “collapse” or saturation. This combination can produce **pole dynamics**: a single concentrated peak in $f$ will first **split** into multiple peaks (due to the shear term), and then each peak’s growth is self-limited by the nonlinear damping. The result is a fragmented twistor representation (multiple poles) corresponding to a decohered state, as well as an overall damping capturing irreversibility.

These toy models are illustrative. In a more realistic twistor evolution equation (to be developed in detail separately), $\mathcal{F}$ might include terms ensuring consistency with spacetime field equations and twistor integrability conditions. For example, one could add coupling to a twistor version of the Poisson equation to account for gravity (so that high $|f|^2$ regions attract each other in twistor space, analogous to gravity causing localization). For now, the key takeaway is that we can **evolve the scalaron’s twistor function $f(Z)$** with an operator $\mathcal{F}$ that includes *linear diffusion* (phase-randomization) and *nonlinear self-interaction* (which can produce branching and effectively model measurement-like collapse). The linear part tends to increase entropy smoothly, while the nonlinear part can cause rapid jumps (e.g. splitting one coherent mode into pieces). A more detailed twistor evolution equation, including all geometric constraints, is left for an Appendix or future work – here we establish the principle with these simplified forms.

**Twistor Entropy Invariants**

To track the “disorder” or complexity of the scalaron in **twistor space**, we define several **entropy-like invariants**. These are quantities derived from $f(Z)$ that remain constant for a perfectly coherent state and increase as the twistor representation becomes more complex (indicating loss of coherence or information spreading). We propose three concrete measures:

* **Pole count ($N\_{\text{poles}}$):** If $f(Z)$ is represented as a rational function or product of factors (for example, $f(Z)$ might be characterized by a set of poles or zeros in twistor space, each corresponding to a basic wave component or topological feature​file-4bzwyu5xwcza2f2huwkyos), one simple invariant is the *number of poles*. We define

Stw(#)  =  ln⁡Npoles ,S\_{\text{tw}}^{(\#)} \;=\; \ln N\_{\text{poles}} \,,Stw(#)​=lnNpoles​,

the logarithm of the pole count. A fully coherent scalaron (one dominant mode) might correspond to $N\_{\text{poles}}=1$ (or a small fixed number), so $S\_{\text{tw}}^{(#)} \approx 0$. As the field decoheres into multiple independent pieces, the twistor function may require multiple poles to represent it. For example, each quasi-coherent domain or each significant wave mode could introduce a new pole in $f(Z)$. Thus $N\_{\text{poles}}$ will grow, and $S\_{\text{tw}}=\ln N\_{\text{poles}}$ increases. This is analogous to counting distinct wave packet components: more components = higher entropy. In gravitational collapse scenarios, $N\_{\text{poles}}$ might jump as the field’s twistor description becomes more intricate (e.g. encoding a black hole might require many twistor components, though ultimately a black hole state might simplify again – see below). **Example:** Start with one pole at $Z\_0$ (pure state). After a mild decoherence, perhaps you have $N\_{\text{poles}}=3$ (the original plus two satellites), so $S\_{\text{tw}}^{(#)} = \ln 3$, indicating increased complexity. If those poles eventually merge or settle (say, a collapse that ends in a single classical object), $N\_{\text{poles}}$ might stop increasing further.

* **Pole weight entropy:** Not only the count, but the **distribution of “weight” among poles** matters. If one pole carries most of $f(Z)$’s residue (dominant contribution) and others are tiny, the state is still nearly coherent. But if many poles contribute comparably, the state is highly mixed. We define weights $\tilde p\_i$ for each pole (for instance, proportional to the residue or contribution of pole $i$ to some norm of $f$) such that $\sum\_i \tilde p\_i=1$. Then

Stw(Shannon)  =  − ∑ip~iln⁡p~i ,S\_{\text{tw}}^{(\text{Shannon})} \;=\; -\,\sum\_i \tilde p\_i \ln \tilde p\_i \,,Stw(Shannon)​=−i∑​p~​i​lnp~​i​,

in direct analogy to Shannon entropy​[en.wikipedia.org](https://en.wikipedia.org/wiki/Von_Neumann_entropy#:~:text=basis%20of%20its%20eigenvectors%20Image%3A,2). This quantity is **low when one pole dominates** (e.g. ${ \tilde p\_1\approx1, \tilde p\_{2,3...}\approx0}$ gives $S\_{\text{tw}}\approx0$) and **maximal when many poles share the weight evenly** (maximizing disorder in twistor space). For instance, if two poles each carry ~50% of the weight, $S\_{\text{tw}}=\ln 2$. If ten poles each carry 10%, $S\_{\text{tw}}\approx \ln 10$ (larger). This invariant is sensitive to how the field’s “quantum amplitude” splits among independent components. **Example:** In a gradual decoherence, the dominant pole’s weight might go from 100% to 90% to 50% as new modes/poles pick up the rest. $S\_{\text{tw}}$ would rise from 0 to a modest value and towards something larger (approaching $\ln N\_{\text{poles}}$ if weights equalize). This tracks decoherence more finely than just pole count – it distinguishes one big + many tiny poles (low entropy) from many equal poles (high entropy).

* **Sheaf patch count / minimal rank:** Twistor representations often require covering the twistor space with coordinate patches and gluing rules (sheafs) to describe global solutions​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos. A **pure coherent state** might be described by a single analytic patch (rank-1 holomorphic solution). As the state gets more complicated (especially if topological defects like branch cuts or multiple disconnected phase domains appear​file-4bzwyu5xwcza2f2huwkyos), one might need multiple patches or a higher-rank bundle to describe $f(Z)$. We can define $S\_{\text{tw}}$ in this context as $\ln N\_{\text{patches}}$ or simply the number of functionally independent sections needed. Equivalently, one can consider the **minimal rank** $r$ of a twistor bundle needed to embed the solution; a single coherent mode is $r=1$, while a superposition of $k$ unrelated modes might require $r=k$. In practice, $N\_{\text{patches}}$ or $r$ will increase as the field decoheres. **Example:** Suppose initially the scalaron’s twistor function is a single simple analytic form on the whole Riemann sphere (one patch). After interference and decoherence produce, say, a phase vortex or discontinuity, no single holomorphic function covers all features – you now need two patches (one excluding the singularity, one around it). Thus $S\_{\text{tw}}$ could jump from $\ln 1 =0$ to $\ln 2$. Further fragmentation into separate domains of analyticity (or additional branch cuts) would require more patches, increasing $S\_{\text{tw}}$. This measure connects to topology: topologically nontrivial configurations (like multiple disjoint phase domains or twistors representing separated entities) demand more complex twistor descriptions.

Each of these invariants gives a way to **track decoherence or collapse in twistor space**. During gradual decoherence, we expect to see a steady rise in $N\_{\text{poles}}$ and in the Shannon-like pole entropy, reflecting the generation of multiple comparable components. During a sudden collapse event, there might be a rapid increase in these measures (as the field’s twistor description becomes highly complex), possibly followed by some simplification if a classical singularity forms (for example, a black hole might be represented by a particular single twistor structure, but the information about how it formed is in the myriad poles that went into it​file-4bzwyu5xwcza2f2huwkyos). In all cases, **$S\_{\text{tw}}$ tends to grow** whenever the scalaron’s state becomes more mixed or information gets dispersed among more degrees of freedom in twistor space.

**Observation Bridge: Entropy & Twistor Dynamics to Observables**

Finally, we connect the growth of entropy and twistor deformation to potential **observable signals** in astrophysics and cosmology. The scalaron’s coherence (or lack thereof) can imprint on several phenomena:

* **Gravitational wave burst “entropy” signature:** A violent event like the collapse of a scalaron core (e.g. forming a black hole or a bosenova-like implosion) will emit gravitational waves. If the scalaron field is highly coherent, the collapse might be symmetric or oscillatory, producing a relatively monochromatic gravitational wave signal (low entropy waveforms). However, if the field is decohered – different parts collapsing out of phase – the gravitational radiation will be more irregular and broadband. In effect, the **gravitational wave burst carries an entropy**: a chaotic, noisy waveform indicates the source had many uncorrelated moving pieces (high $S(t)$), whereas a clean waveform suggests a coherent, low-entropy source. By analyzing gravitational wave spectra from events (e.g. core collapses or mergers in fuzzy dark matter halos), one could infer the scalaron’s entropy. For instance, a **burst with a very spread-out frequency content** or with random amplitude modulations might correspond to a high $S\_{\mathrm{vN}}$ and multiple twistor poles participating in the motion. On the other hand, a burst with a dominant frequency mode (like a single ringing mode) might indicate the scalaron field remained relatively coherent up to the collapse. This way, gravitational wave observations act as a **probe of the twistor entropy** – the more complex the twistor structure of the event, the more “complex” (in information content) the emitted waves.
* **Lensing flicker suppression (interference loss):** Fuzzy/adaptive dark matter can produce time-dependent gravitational lensing effects. When the scalaron field is coherent, interference patterns in the density can cause **rapid oscillations in the lensing potential** – for example, a distant quasar lensed by a fuzzy dark matter halo might exhibit high-frequency brightness fluctuations (“flicker”) due to moving interference fringes in the halo. As decoherence sets in, these interference fringes are washed out. The scalaron behaves more like classical particles, yielding a steadier gravitational potential. Thus, **increasing $S(t)$ suppresses lensing flicker**. Observationally, one could compare young, relatively coherent halos (which might show subtle coherent lensing oscillations) to older or denser halos (which should show none of that rapid flicker because the phases have randomized). In our framework, a high twistor entropy $S\_{\text{tw}}$ (many incoherent patches/poles) corresponds to each lensing “speckle” contributing independently, which averages out fast. Conversely, a low entropy coherent field could, in principle, produce observable quasi-coherent diffraction patterns in lensing (an exciting but challenging signal to detect). Upcoming high-cadence lensing observations or pulsar-timing arrays passing through dark matter filaments could set limits on such flicker, thereby constraining the scalaron’s coherence over time.
* **Mode-count growth in $P(k)$ from scalaron decoherence:** As the scalaron decoheres, it populates a broader range of momentum modes. In the early (coherent) stage, most of the dark matter mass is in the condensate mode (low-$k$ momentum) with only mild quantum fluctuations at specific scales (like the de Broglie wavelength scale). The matter power spectrum $P(k)$ in such a state might have a sharp cutoff at high $k$ (small scales) due to quantum pressure suppressing small structure. However, **decoherence feeds power into many modes**: essentially the field “heats up” or fragments, filling out a **broader $P(k)$**. One can think of this as the wavefunction breaking into many wavelets (granules​file-3zh15rq3mb1bnnjszwe2yx), each with its own phase – effectively adding fine-grained structure. Thus, one expects an **increase in the small-scale power** as decoherence progresses. For example, a simulation might show that at time $t\_0$ (coherent), $P(k)$ beyond some $k\_{\rm peak}$ is very low, but by time $t\_1$ (decohered), there is a substantial high-$k$ tail (more small clumps, akin to classical shot noise). The **mode-count** (number of modes significantly populated) can be quantified by, say, the occupation number spectrum or spectral entropy in $k$-space, which is directly related to $S\_{\mathrm{vN}}$. Observationally, one could compare the small-scale structure of dark matter at different cosmic times: a young universe (if the scalaron was still coherent) might have smoother small-scale density than expected, whereas today’s universe (with higher $S(t)$) has more substructure. If future surveys find an excess of small-scale power that increases with time or environment (beyond what a fixed fuzzy dark matter model predicts), it could be a hint of ongoing decoherence. In summary, **the growth of $S(t)$ corresponds to a transfer of power to higher $k$ modes** in the matter spectrum – effectively an entropy-driven enhancement of structure on small scales.

These bridges between theory and observation will be developed further in RFT 9.8. The key point is that the abstract concepts of scalaron entropy and twistor complexity are not just mathematical curiosities – they have real consequences for what we can measure. Gravitational wave detectors, lensing observatories, and large-scale structure surveys each offer a window into the quantum state of dark matter. By linking $S(t)$ and $S\_{\text{tw}}$ to concrete signatures (waveform entropy, flicker/no-flicker in lensing, evolution of $P(k)$), we prepare a path to **test the adaptive scalaron framework** experimentally. An observed trend of increasing disorder in these signals over time would lend support to the idea that the scalaron field is progressively decohering, exactly as our entropy formalism describes.

***(Optional Appendix: Simulation Pseudocode for $F\_c$ and $S(t)$)***

For completeness, we provide a minimal pseudocode sketch showing how one might compute the coherence fraction field $F\_c(x,t)$ and entropy $S(t)$ in a Schrödinger–Poisson (SP) simulation of the scalaron:

python

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# Assume we have the scalar field wavefunction psi(x) on a grid at each timestep.

# Also assume functions to get one-particle density matrix or to diagonalize it.

for each time step t:

# Compute local density field

rho = abs(psi)\*\*2 # rho(x,t) on the grid

# Option A: Phase-correlation based F\_c

F\_c = array\_of\_same\_shape\_as(rho)

for each grid point x:

# compute coherent amplitude via phase correlation integral (e.g., global overlap)

C\_x = 0

for each grid point y:

C\_x += psi[x].conjugate() \* psi[y] \* dV # integrate psi\* at x with psi at y

C\_x = C\_x / total\_mass # normalize by total mass M

F\_c[x] = abs(C\_x) / (rho[x] / total\_mass) # |coherent amplitude| / (rho\_x/M)

# (In practice, one might integrate only over a coherence length around x for efficiency)

# Option B: Eigenmode (Penrose-Onsager) based F\_c

rho\_matrix = compute\_one\_particle\_density\_matrix(psi) # rho1(x,x') = psi^\*(x)psi(x')

eigenvalues, eigenmodes = diagonalize(rho\_matrix) # get {n\_i, phi\_i(x)}

p0 = eigenvalues[0] / total\_mass # dominant eigenvalue fraction

phi0 = eigenmodes[0] # dominant eigenfunction

for each grid point x:

F\_c[x] = (p0 \* abs(phi0[x])\*\*2) / rho[x] # fraction of density from mode 0

# Compute scalaron entropy S(t)

S = 0.0

for each grid point x:

if F\_c[x] > 0: # avoid log(0)

S += - rho[x] \* log(F\_c[x]) \* dV # accumulate -ρ ln F\_c over space

record(S, t)

In this pseudocode, Option A and Option B are two ways to get $F\_c(x,t)$ (one can choose one based on practicality). The integration volume element is $dV$ (for a grid cell). In a real code, one would vectorize operations and use FFTs or linear algebra libraries for efficiency (for instance, diagonalizing $\hat{\rho}*1$ for a large grid is expensive, so one might approximate the dominant eigenmode via iterative methods). The end result each step is an updated entropy $S(t)$. This can be plotted to verify that $\dot S(t) \ge 0$ (entropy should increase or stay constant, reflecting the second law in this context). If one also tracks the eigenvalues $p\_i(t)$, one could similarly compute $S*{\mathrm{vN}}(t)$ and check its evolution against $S(t)$.

*(The above pseudocode is highly simplified and assumes access to the full wavefunction. In a simulation that only stores densities and phases, one might estimate $F\_c$ by tracking phase fields or coherence lengths instead. Nonetheless, it provides a blueprint for implementing the theoretical definitions in a numerical experiment.)*

***(Optional Appendix: Monotonic Increase of Twistor Entropy)***

One important consistency check is that our twistor entropy measures obey an $H$-theorem: namely, **$dS\_{\text{tw}}/dt \ge 0$** under the twistor evolution (absent external pumping of coherence). While a rigorous proof requires the full twistor dynamics, we can sketch why this should hold for the simplified models:

Consider the **linear diffusion model** $\partial\_t f = D \nabla\_Z^2 f - \Gamma f$ introduced above. If we interpret $P(Z,t) = |f(Z,t)|^2$ (suitably normalized) as a distribution in twistor space, this equation tends to **spread out** $P(Z)$. Absent the sink term $-\Gamma f$ (which uniformly damps but doesn’t add order), pure diffusion in an isolated system maximizes entropy. In fact, one can show for the analogous probability diffusion equation $\partial\_t P = D \nabla^2 P$ that the **Shannon entropy** $S[P]=-\int P \ln P,dZ$ *increases* with time. Intuitively, diffusion homogenizes $P$, moving it toward a more uniform (higher entropy) distribution. A short derivation:

dSdt=−∫(∂tP) ln⁡P dZ=−D∫∇2P ln⁡P dZ .\frac{dS}{dt} = -\int (\partial\_t P)\,\ln P\,dZ = -D\int \nabla^2 P \,\ln P\,dZ\,.dtdS​=−∫(∂t​P)lnPdZ=−D∫∇2PlnPdZ.

Integrating by parts twice (and assuming $P$ and $\nabla P$ vanish at the twistor-space boundary or infinity), this becomes

dSdt=D∫∣∇P∣2P dZ ≥ 0 ,\frac{dS}{dt} = D \int \frac{|\nabla P|^2}{P}\,dZ \,\ge\,0\,,dtdS​=D∫P∣∇P∣2​dZ≥0,

since $|\nabla P|^2/P \ge 0$ everywhere. Thus diffusion alone gives $dS/dt \ge 0$, with equality only for $P$ uniform (already maximum entropy). The damping term $-\Gamma f$ simply multiplies $P$ by a decaying factor, which does not introduce any new structure (it can only increase relative importance of uniform background as structured parts decay). Hence, in the linear model, **$S\_{\text{tw}}$ (measured, say, by $-\int |f|^2 \ln|f|^2$ or by the pole entropy if we track modes) is non-decreasing**. This is completely analogous to the von Neumann entropy of a density matrix increasing under Lindblad decoherence dynamics in open quantum systems​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos.

For the **nonlinear model**, the analysis is more complex, but the physical expectation of irreversibility remains. The pole-splitting term (shear in twistor space) redistributes amplitude among multiple locations, and the $-|f|^2 f$ term quickly damps coherent high-amplitude features. Neither effect is invertible without fine tuning (e.g. once a pole splits into two and their phases decohere, they won’t spontaneously re-merge in exact phase). In a coarse-grained sense, one can imagine a kinetic equation for the weights $\tilde p\_i(t)$ of each pole. It would involve poles exchanging weight or spawning new ones, but because each interaction is like a scattering that randomizes phases, the detailed balance should favor **equipartition of weight** among available modes over time. Equipartition is the maximum entropy state (all $\tilde p\_i$ equal). Thus, one expects $-\sum\_i \tilde p\_i \ln \tilde p\_i$ to increase or stay constant. Even the pole count $N\_{\text{poles}}$ is unlikely to decrease: once multiple poles exist, a reversal (many poles collapsing back into one) would require them to arrive with exactly correlated phases – a measure-zero probability in a chaotic system. More formally, the twistor evolution $\mathcal{F}$ we propose is analogous to a **coarse-graining** or a semigroup evolution in an information space, not a time-reversible Hamiltonian. It should obey an $H$-theorem, ensuring the **arrow of time** in twistor space aligns with increasing $S\_{\text{tw}}$, consistent with the entropy increase in spacetime ($S(t)$)​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos.

In summary, under reasonable assumptions each of our twistor entropy indicators either stays constant or grows with time. This cements the interpretation of $S\_{\text{tw}}$ as an entropy: it increases as the scalaron field’s twistor-state becomes more “mixed” or complex. Verifying $\dot S\_{\text{tw}}>0$ in full generality will be part of future analytical work, but all evidence from these toy models and physical reasoning supports it.